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EFFECT OF THE BOUNDARY LAYER ON THE BASE PRESSURE IN A TWO-DIMENSIONAL FLOW WITH A MACH NUMBER M = 5

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New data on the base pressure in a two-dimensional flow with a Mach number M = 5 *are obtained for a wide range of variation of the normalized boundary-layer thickness in the flow-separation cross section. The test results are compared with Tanner's theory, and a conclusion is made that this numerical model has to be corrected.*

Key words: *supersonic flow, turbulent boundary layer, base pressure.*

Introduction. The dimensionless base pressure p_b/p_e of the body in a compressible flow has a strong effect on the near-wake structure. The problem of the near wake and, in particular, the base drag is discussed in numerous experimental and theoretical studies. A large list of papers on this topic can be found in the review [1] and monographs [2–4]. Various experiments and simplified numerical models are used to study the dependence of the base pressure of bodies with different configurations on the basic governing parameters, which is usually presented in the form $p_b/p_e = f(\mathrm{M}_e, \alpha, \delta/(t/2)$ or $\delta^{**}/(t/2))$, where M_e is the free-stream Mach number, δ is the boundary-layer thickness, δ^{**} is the momentum thickness, $t/2$ is the half-height (radius) of the base cross section, and α is the angle of deviation of the body contour from the free-stream direction in the flow-separation cross section. The most extensively examined range is $\mathrm{M}_e \leqslant 3$, $\delta/(t/2) \leqslant 1$; for this combination of parameters, the data available in the literature are sufficient to determine the base drag of bodies of simple geometry (cone, cone–cylinder, wedge, and step).

In the range of high supersonic velocities ($\mathrm{M}_e > 3$), the measurements were performed either with a fixed ratio $\delta/(t/2)$, or the range of variation of the dimensionless boundary-layer thickness was varied within narrow limits corresponding to $\delta^{**}/(t/2) \leqslant 1$. The latter, in particular, does not allow us to determine the accuracy of taking into account the influence of this parameter on the base pressure in existing numerical models. Note that large values of $\delta/(t/2)$ are reached in some situations important for practice, e.g., in the flow around extended bodies or at low Reynolds numbers.

It follows from the above-given information that the study of the boundary-layer effect on the base drag is still important. This problem is discussed in the present paper, which describes detailed measurements of the base pressure on the edge of a two-dimensional body in a supersonic flow with a Mach number M = 5 in a wide range of variation of the boundary-layer thickness at the exit, up to $\delta/(t/2) = 3.2$. The experimental data obtained are used to test Tanner's numerical model [5–7], which, in contrast to other known numerical methods (see, e.g., [1–4]) claims that it can predict the base drag for bodies of different geometries for arbitrary values of M and $\delta/(t/2)$, and which was previously tested at lower Mach numbers.

Setup, Models, and Measurement Technique. The base pressure was measured at the edge of a nozzle blade in the course of extensive studies of the flow structure formed by a block of two-dimensional supersonic nozzles [8] (as applied to the problem of optical quality of the active medium of the gas-dynamic laser [9, 10]). The test conditions and the model geometry were suitable to perform the study with substantial variation of the dimensionless boundary-layer thickness in the flow-separation cross section.

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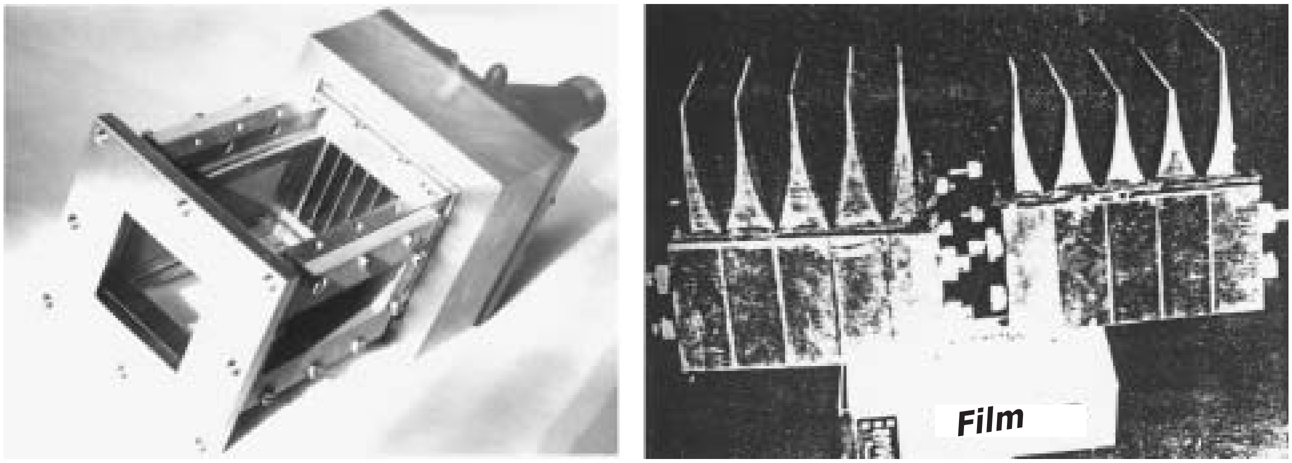

Fig. 1. Experimental setup (a) and nozzle block (b).

The experiments were performed in a wind tunnel with a (51 × 56)-mm test section. The main elements of the setup were the settling chamber with an equalizing grid, a constant-cross-section channel, and an ejector. A photograph of a settling chamber with the attached block of nozzle blades and part of the test section is shown in Fig. 1a. In probing tests, the optical windows were replaced by Plexiglas windows with orifices intended for probes; the probes were moved by a traversing gear equipped by a micrometer.

The setup was fed by purified air from high-pressure gas holders. The pressure in the settling chamber during the experiments was $p_0 = 0.75$–3.75 MPa. The Mach number in the cross section of the edge of the nozzle blades was close to the design value M = 5. In most experiments, the air fed into the settling chamber was heated in an electric heater up to a temperature $T_0 = (340 \pm 3)$ K, which ensured the absence of condensation.

The experimental models were blocks of two-dimensional contoured supersonic nozzles (Fig. 1b) located at the test-section entrance. Several variants of nozzle blades with an identical contour were made; the only parameter being varied was the thickness of the nozzle edge t. The supersonic contour of the nozzle with a corner point was calculated by the method of characteristics for the Mach number M = 5 in the exit cross section, and then it was corrected for the boundary-layer displacement thickness on the wall. As a result, for the throat height $h^* = 0.49$ mm, the exit cross-section height was $H = 13.45$ mm, the length of the supersonic part was $L = 37$ mm, and the blade contour at the nozzle exit was parallel to the centerline.

For the experiments, we prepared the central blades of two blocks (the edge thickness of the prepared blades was $t = 0.70$ and 2.3 mm, respectively). Pressure orifices 0.3 mm in diameter were drilled in the edges of these blades; with some extra channels drilled at an angle of 90°, these orifices were connected to measuring lines.

The stagnation parameters (p_0, T_0) and the pressure at the edge of the nozzle blade (base pressure p_b) were measured in the experiments. The distribution of flow parameters in the nozzle cross section at a distance of 2 mm from the exit was determined from the results measured by total and static pressure probes. The pressure in the flow at the nozzle exit (p_e) was found from the static pressure measured in this cross section with allowance for a minor expansion of the nozzle. The Pitot tube had a flattened intake 0.3 × 1 mm. The diameter of the static pressure probe (with an ogival tip) was 1 mm. The pressure was registered by IKD 27 DF transducers (with nominal values of 0.016, 0.06, and 0.1 atm and measurement accuracy of 3%).

Based on the results measured in the boundary layer in the vicinity of the nozzle exit, we calculated the momentum thickness δ^{**}. The parameter $\delta^{**}/(t/2)$ was varied by changing the Reynolds number at the nozzle exit [$\mathrm{Re}_L = (1$–$5) \cdot 10^6$] and the nozzle-edge thickness ($t = 0.75$ and 2.3 mm). Thus, the range $\delta^{**}/(t/2) = 0.028$–$0.127$ was examined in the experiments. The ratio of the boundary-layer thickness to the half-height of the nozzle edge was varied in the interval $\delta/(t/2) = 0.7$–3.2, i.e., the test conditions corresponded to the case of a thick boundary layer.

Experimental Results and Analysis. The schlieren picture of the flow downstream of the nozzle-blade edge of thickness $t = 2.3$ mm is shown in Fig. 2. Such a flow structure has been discussed in the literature and

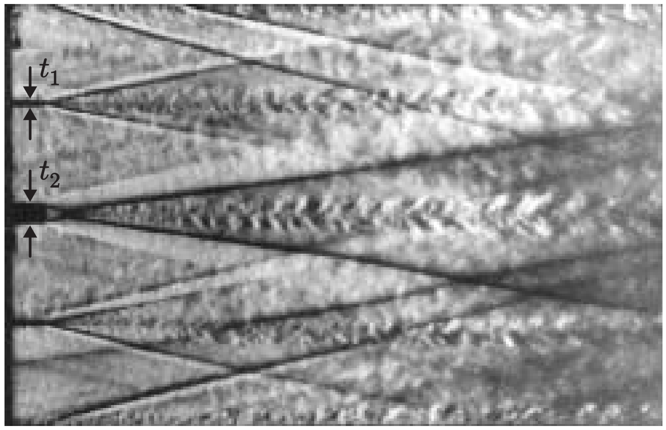

Fig. 2. Schlieren picture of the flow behind the nozzle block.

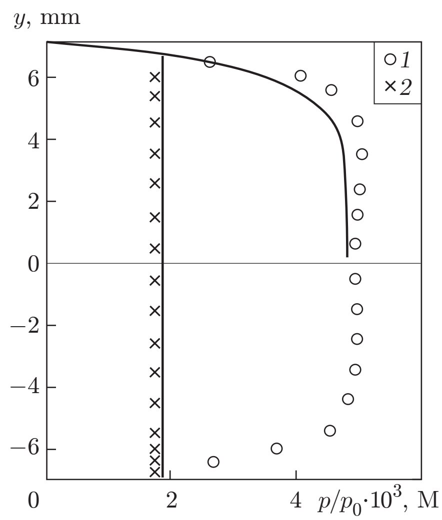

Fig. 3

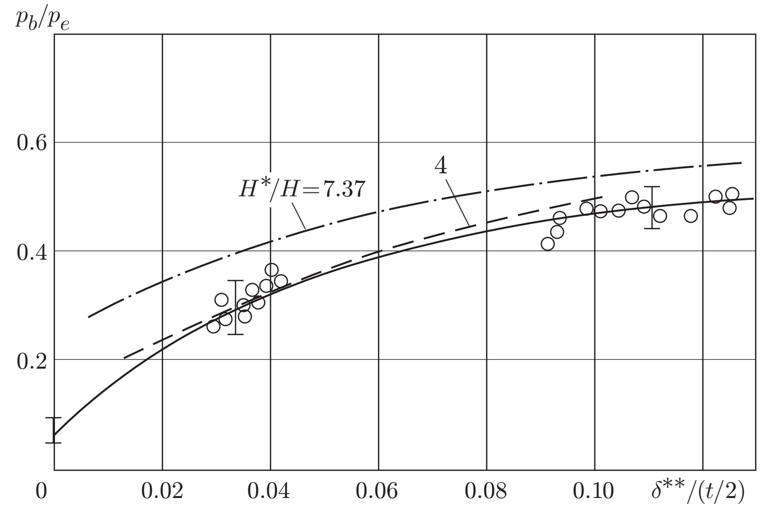

Fig. 4

Fig. 3. Distribution of flow parameters in the vicinity of the nozzle exit for $\mathrm{Re}_L = 1.9 \cdot 10^6$: the points 1 and 2 refer to the experimental data for M(y) and $P(y)$, respectively, and the curves refer to the results of calculations.

Fig. 4. Base pressure versus the boundary-layer momentum thickness: the points show the experimental data; the solid curve is the approximation of experimental data with extrapolation of the experimental dependence to the zero boundary-layer thickness; the dot-and-dashed and dashed curves show the calculations by Tanner's model with $H^*/H = 7.37$ and 4, respectively; the vertical bars correspond to the maximum error of the quantity p_b/p_e.

does not need particular comments. Only the shock wave emanating from the corner point and located below the expansion fan is worth noting. Obviously, this is the so-called "lip shock" examined in detail in [11]. After reflection from the centerline of the base region, it coalesces with the tail shock.

Since the main goal of the present work was to study the influence of the boundary layer on the base pressure, much attention was paid to determining the momentum thickness.

Figure 3 shows the Mach number and static pressure profiles in the flow in the vicinity of the nozzle exit (the coordinate $y = 0$ corresponds to the channel centerline). The velocity distribution across the boundary layer obeys a power law with an exponent of 1/7.

Figure 4 shows the dimensionless base pressure p_b/p_e as a function of the dimensionless momentum thickness $\delta^{**}/(t/2)$ near the flow-separation point. Note, the quantity $\delta^{**}/(t/2)$ is commonly used as a governing parameter, though this has not been adequately justified. Two groups of points in Fig. 4 refer to experiments with different edge thicknesses. The character of the dependence of the base pressure on the boundary-layer thickness obtained in the present work is in agreement with the results of the previous studies performed for lower Mach numbers, though the boundary-layer effect on the value of p_b/p_e is more pronounced.

To demonstrate this fact, we compare the experimentally obtained values of p_b/p_e with the corresponding values for a vanishingly small boundary-layer thickness. Available theoretical models predict substantially different values of p_b/p_e as $\delta/t \to 0$. Thus, according to the results on $p_b/p_e = f(\delta/t)$ calculated in [5] and extrapolated toward higher Mach numbers, we obtain $p_{b,0}/p_e \approx 0.03$–0.09 for $\delta/t \to 0$ and M = 5. The limiting values of this range are marked in Fig. 4 by dashes on the vertical axis. Assuming the mean value to be $p_{b,0}/p_e = 0.06$ for $\delta/t \to 0$ and M = 5 and comparing it with the maximum value, we obtain $p_{b,\max}/p_{b,0} \approx 8$ for the present test conditions. At the same time, according to data obtained in [3] for M = 1.5 and 3 in the range $\delta/(t/2) = 0.05$–1.0, the values of p_b/p_e are higher by a factor of 1.2 and 2, respectively.

Let us now consider how our measurement results agree with calculations by Tanner's theory [6]. This model was chosen for comparisons with experimental data owing to its universality. In [6], Tanner derived relations for determining the drag coefficient of axisymmetric and two-dimensional bodies of different geometry for subsonic, transonic, and supersonic flow velocities with allowance for the boundary-layer effect. Based on the data given in [6], the calculated results offer a fairly accurate description of experimentally obtained dependences of the drag coefficient on the Mach number and boundary-layer thickness.

Yet, the model was mainly tested against experimental results in the range of Mach numbers $M \leqslant 3$. The calculations were compared with experimental data for a number of two-dimensional configurations for M = 3–7 in [7], but this was done only for one value of the dimensionless momentum thickness $\delta^{**}/(t/2) = 0.045$. The tests were performed in a shock tube, and the boundary-layer thickness at the nozzle exit was not measured but was calculated by formulas for an incompressible fluid, which is responsible for some uncertainty in interpreting the measurement results. In turn, the data of the present work allow testing Tanner's model with such a combination of the governing parameters (M = 5 and a wide range of momentum thicknesses) in which the capabilities of this model to predict the base pressure have not been analyzed before.

Tanner's theory involves the known Oswatisch's relation [12] for the flow entropy and drag of the body, as well as some assumptions introduced by the author. The main assumption is that the base pressure is identical for viscous and inviscid flows. Analyzing the backward-facing step (the case that is almost identical to the present test conditions), Tanner postulated the velocity distribution in the mixing layer in the cross section immediately behind the flow-reattachment point and assumed that the pressure in the separation region was constant. The flow outside the mixing layer was assumed to be isentropic. The angle of inclination of the tail shock to the surface was assumed to be identical in both cases (viscous and inviscid flows).

After that, the change in the entropy flux behind the reattachment point for the cases considered is analyzed. Equating these fluxes, one can determine the single value of the base pressure. The relation for the entropy flux in the inviscid flow contains the parameter H^*, which is the "effective" distance from the wall (axis of symmetry) to the point where weak (isentropic) compression waves merge to form the tail shock; the relations for the viscous flow contain the mixing-layer thickness H in the cross section considered. Both quantities appear in the resultant formula as the ratio H^*/H. The value of the parameter $H^*/H = 7.37$ was determined by comparing the theoretical and experimental values for M = 1.73 and a thin boundary layer. After that, the boundary-layer effect was taken into account by supplementing the drag of the body by its friction drag characterized by the momentum thickness at the separation point.

It follows from the above-described considerations that Tanner's theory is not sufficiently rigorous (though all previous models were not rigorous either), and the area of its application can be determined on the basis of a detailed comparison with experimental data.

The results calculated by Tanner's model for the present test conditions are plotted in Fig. 4. The path of the calculated curve coincides qualitatively with the dependence obtained in experiments. For a thick boundary layer, the quantitative agreement is also fairly good [the difference is within 10% for $\delta^{**}/(t/2) \geqslant 0.1$]. As the parameter $\delta^{**}/(t/2)$ decreases, the numerical curve moves away from the experimental points. For $\delta^{**}/(t/2) = 0.03$, for example, the theoretical value of the base pressure is 25% higher than the experimental one; the difference becomes even greater as $\delta^{**}/(t/2) \to 0$.

We performed calculations by Tanner's model, the parameter H^*/H being varied. In this connection, it should be noted that it does not follow from the theory that the parameter H^*/H is constant for different values of Mach number. Magi and Gay [13] drew Tanner's attention to this circumstance, but Tanner noted in [7] that the experimental data used in [13] for comparisons with the theory are not accurate enough. It is seen from Fig. 4, nevertheless, that the calculations by Tanner's model with the parameter $H^*/H = 4$ are in better agreement with the experiment for M = 5 than those based on the value $H^*/H = 7.37$ recommended by Tanner and derived from a comparison with experimental data for M < 3. Hence, the dependence of the parameter H^*/H on the Mach number in the model considered should be refined by comparing the calculation results with more reliable experimental data.

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